

Flavia-Corina MITROI-SYMEONIDIS
Eleutherius SYMEONIDIS

LINEAR ALGEBRA **for Economists**

Colecția
Matematici pentru economiști

Editura ASE
București
2024



Copyright © 2024, Editura ASE

Toate drepturile asupra acestei ediții sunt rezervate editurii.

Editura ASE

Piața Romană nr. 6, sector 1, București, România

cod 010374

www.ase.ro

www.editura.ase.ro

editura@ase.ro

Descrierea CIP a Bibliotecii Naționale a României

MITROI-SYMEONIDIS, FLAVIA-CORINA

Linear algebra for economists / Flavia-Corina Mitroi-Symeonidis,

Eleutherius Symeonidis. – București : Editura ASE, 2024

Conține bibliografie

ISBN 978-606-34-0533-4

I. Symeonidis, Eleutherius

51

33

Editura ASE

Copertă: Claudia-Marinela Dumitru

Autorii noștri își asumă întreaga responsabilitate pentru ideile exprimate, pentru originalitatea materialului și pentru sursele bibliografice menționate.

Contents

Preface	7
1 Vector Spaces	9
1.1 Notation and examples	9
1.2 Vector Subspaces	12
1.3 Linear combinations	16
1.4 Bases	17
1.5 Pivot Method (Gauss-Jordan Method)	21
1.6 The Quotient Space	27
1.7 Sums and direct sums	29
2 Linear Operators	37
2.1 Definition. Kernel and Image. Injectivity, surjectivity, bijectivity of linear operators	37
2.2 Isomorphism theorems	42
2.3 The matrix of a linear operator. The change of the bases. The four fundamental spaces.	44
2.4 Matrix similarity. Eigenvalues, eigenvectors, eigenspaces. Char- acteristic polynomial. Diagonalization of endomorphisms on finite- dimensional vector spaces	52
2.5 Jordan Canonical (Normal) form of an endomorphism	69
2.5.1 Finding a Jordan Basis for matrices with bigger sizes . . .	75
2.5.2 Examples of Jordan Bases, for smaller sizes of the matrices	77
2.6 Homogeneous linear systems of first order ODE (ordinary differ- ential equations) with constant coefficients	92
2.6.1 Non-homogeneous linear systems of first order ODE with constant coefficients	102
3 Linear functionals (forms) and dual spaces	105
3.1 Bilinear functionals and sesquilinear functionals	108
3.2 Quadratic forms	116
3.2.1 Jacobi's method of the leading principal minors	120
3.2.2 Gauss' method of diagonalization by completion of squares	125
4 Euclidean spaces	131
4.1 Least squares method	142
4.2 Gram-Schmidt orthogonalization	146
4.3 Orthogonal diagonalization	149

5	Linear operators on inner product spaces	159
5.1	The adjoint operator	159
5.2	Self-adjoint (Hermitian or symmetric) endomorphisms	161
5.3	Orthogonal operators	163
	Bibliography	167

Preface

The concepts and techniques of linear algebra are nowadays widely used in every field of economics. Our book provides a concise introduction to basic topics of linear algebra, being designed to provide many detailed examples, solved exercises, in a natural connection of its chapters, with emphasis on vector spaces and linear operators. It covers the topics of the one-term undergraduate Linear Algebra course at the *Bucharest University of Economic Studies*. In terms of prerequisites, the book assumes a basic background on topics like sets, functions, matrices and determinants (we recommend on those subjects the books [9], [19], [28]).

In Chapter 1 we introduce the vector spaces and their properties, we list several examples and counterexamples, followed by notions like linear independence, bases, spanning sets in the framework of finite dimensional vector spaces. We only consider real and complex vector spaces.

Linear operators are introduced in Chapter 2, with the associated fundamental spaces. The diagonalization of endomorphisms, the algorithm to determine the Jordan Canonical Form of a representation matrix are the goal of the second chapter, which ends with applications to ODE systems.

In Chapter 3 we present the linear, bilinear, sesquilinear functionals. The related quadratic forms are introduced together with two methods to diagonalize them: the Jacobi method (leading principal minors) and the Gauss method (completion of squares). Later, in Chapter 4, in the framework of Euclidean spaces, the orthogonal diagonalization of quadratic forms is presented using the Gram-Schmidt algorithm.

The last chapter deals with linear operators on inner product spaces, namely the adjoint, self-adjoint and orthogonal operators.

This book contains a large number of exercises. The solutions of the exercises are often provided, or we indicate how to solve them. A few are left to the reader: have fun!

The authors

October 2024, Bucharest